AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Distance $= \int_0^6 \lvert v(t) \rvert\,dt = 12.573$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

1A,

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

The speed is increasing at $t=5.5$ because $v(5.5)<0$ and $a(5.5)<0$

$v(5.5) = -0.453$

$a(5.5) = -1.358$

Work for problem 1(b)

$\frac{1}{6-0}\int_0^6 v(t)\,dt = 1.949$

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Continue problem 1 on page 5.

1 A_2

Work for problem 1(c)

$\int_0^6 |v(t)|\,dt = 12.573$

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Work for problem 1(d)

$v(t) = 0$ at $t = 5.1955223$ and $v(t)$ changes signs at $t = 5.1955223$

$X(5.1955223) = 2 + \int_0^{5.1955223} v(t)\,dt$

position at $t = 5.1955223$ is 14.134

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1B,

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

a) The speed of the particle at $t=5.5$ in increasing as both $v(t)$ and $a(t)$ are negative, so $v(t)$ is getting faster in the negative direction. In other words

speed $= |v(t)|$

$|v(t)|$ is increasing

Work for problem 1(b)

$$\frac{v(6)+v(0)}{6-0} = .289$$

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Continue problem 1 on page

Work for problem 1(c)

$v(t) = 2\sin(e^{t/4}) + 1$

1B2

$\int_0^6 |v(t)|\,dt = 12.573$

Work for problem 1(d)

$x(5.196) = \int_0^{5.196} (v(t)) + 2 = 14.135$

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1C1

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

At time $t = 5.5$, the speed of the particle is increasing, because the acceleration and velocity of the particle at time $t = 5.5$ are both negative.

Work for problem 1(b)

Average Velocity $= \frac{v(6) - v(0)}{6 - 0} \approx \boxed{-0.453\ u/t}$

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Continue problem 1 on page

1C2

Work for problem 1(c)

Total Distance Traveled $= 2 + \int_0^6 |v(t)|dt = 2 + 12.573 = \boxed{14.573 \text{ u}}$

We must take the absolute value of $v(t)$ to find the total distance traveled.

Work for problem 1(d)

The particle changes direction when velocity, $v(t)$, changes sign. On the interval $(0,6)$, velocity changes sign from positive to negative once at time $t = 5.196$. Therefore, the particle changes its direction once at time $t = 5.196$.

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AP® CALCULUS AB
2011 SCORING COMMENTARY

Question 1

Overview

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 6$. The position, $x(t)$, of the particle is unknown, but velocity and acceleration functions, $v(t)$ and $a(t)$, respectively, are provided. Part (a) asked students whether the speed of the particle is increasing or decreasing at time $t = 5.5$. Students should have evaluated both the velocity and the acceleration functions at $t = 5.5$; because $v(5.5) < 0$ and $a(5.5) < 0$, the particle's speed is increasing. Part (b) asked for the average velocity of the particle during the given time interval. This can be computed as an average value, $\frac{1}{6-0}\int_0^6 v(t)\,dt$, and evaluated on a calculator. Part (c) asked for the total distance traveled by the particle. The total distance is the value of $\int_0^6 |v(t)|\,dt$, which can be computed directly on the calculator, or by splitting the interval into a segment on which $v(t) > 0$ and one on which $v(t) < 0$, and then appropriately combining the corresponding definite integrals of velocity. Part (d) highlighted that the particle changes direction exactly once during the interval, and asked for the position of the particle at that time. If they had not already done so, students should have used their calculators to find the solution to $v(t) = 0$ with $0 \le t \le 6$. If the solution is $t = A$, the position of the particle at that time is then calculated as $x(A) = 2 + \int_0^A v(t)\,dt$.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In parts (a) and (c) the student's work is correct. In part (b) the student's work is incorrect. In part (d) the student does not write $v(t) = 0$, so the first point was not earned. The student earned the integral and answer points.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student finds average acceleration, so no points were earned. In part (c) the student earned 1 point for the integral. Because the student incorrectly adds 2, the answer point was not earned. In part (d) the student's phrase "velocity, $v(t)$, changes sign" earned the first point. The student finds the value $t = 5.196$ but does not continue.

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature $100°\text{C}$ were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a) $H'(3.5) \approx \dfrac{H(5) - H(2)}{5 - 2}$

$= \dfrac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2\cdot\frac{66+60}{2} + 3\cdot\frac{60+52}{2} + 4\cdot\frac{52+44}{2} + 1\cdot\frac{44+43}{2}\right)$$
$$= 52.95$$

3 : { 1 : meaning of expression
1 : trapezoidal sum
1 : estimate }

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : { 1 : value of integral
1 : meaning of expression }

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : { 1 : integrand
1 : uses $B(0) = 100$
1 : answer }

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2A1

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Work for problem 2(a)

$$\frac{H(5)-H(2)}{5-2} = \boxed{\frac{-8}{3}\ \frac{°C}{min}}$$

Work for problem 2(b)

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{\left[2\left(\frac{66+60}{2}\right)+3\left(\frac{52+60}{2}\right)+4\left(\frac{44+52}{2}\right)+1\left(\frac{43+44}{2}\right)\right]}{10} = \boxed{52.95}$$

THIS ~~FX~~ REPRESENTS THE AVERAGE TEMPERATURE IN DEGREES CELSIUS OF THE TEA ~~O~~ OVER THE INTERVAL $0 \le t \le 10$

Continue problem 2 on page 7.

2A2

Work for problem 2(c)

$$\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = \boxed{-23°C}$$

THIS EXPRESSION SHOWS THE TOTAL CHANGE IN TEMPERATURE IN DEGREES CELSIUS FROM ~~[illegible]~~ $t=0$ to $t=10$.

Work for problem 2(d)

$B'(t) = -13.84e^{-.173t}$

$B(10) = \int_0^{10} -13.84e^{-.173t} + 100 = 100 - 65.817 = 34.1827$

~~[illegible]~~

$43 - 34.1827 = 8.817$ ~~[illegible]~~

$= \boxed{8.817°C \text{ cooler}}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2B1

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Work for problem 2(a)

$H'(3.5) = \frac{H(5) - H(2)}{5-2} = \frac{52-60}{3} = \boxed{\frac{-8}{3}\,^{\circ}C/min}$

Work for problem 2(b)

$\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average value that $H(t)$ is decreasing per min on the interval $0 \leq t \leq 10$

$\frac{1}{10}\left(f(0) + 2f(2) + 2f(5) + 2f(9) + f(10)\right)$

$\frac{1}{10}\left(66 + 2(60) + 2(52) + 2(44) + 43\right)$

$\frac{1}{10}\left(66 + 120 + 104 + 88 + 43\right)$

$\frac{1}{10}(421) = \boxed{42.1 \approx \frac{1}{10}\int_0^{10} H(t)\,dt}$

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Continue problem 2 on page 7.

2B

Work for problem 2(c)

$\int_0^{10} H'(t)\,dt = H(t)\Big]_0^{10} = H(10) - H(0)$

$43 - 66 = \boxed{-23°C}$

This is the total change in temperature of the tea pot on the interval $0 \le t \le 10$

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Work for problem 2(d)

$\int_0^{10} B'(t)\,dt = -65.8172$

$100 - 65.8172 = 34.1827$

$43 - 34.1827 = \boxed{8.8172}$

The biscuits are 8.8172°C cooler than the tea.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2C1

Work for problem 2(a)

$\frac{52-60}{5-2} = -8/3$ °C/min

Work for problem 2(b)

$\frac{1}{10}\int_0^{10} H(t)$ is the average rate of change in temperature for 0 to 10 minutes.

$\frac{1}{2}h(b_1+b_2)$

$\frac{1}{10}\left(\frac{1}{2}(2)(66+60)+\frac{1}{2}(3)(60+52)+\frac{1}{2}(4)(52+44)+\frac{1}{2}(1)(44+43)\right)$

$\frac{1}{10}(529.5)$

$\frac{1}{10}\int_0^{10} H(t)\,dt \approx 52.95$

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Continue problem 2 on page 7.

2C2

Work for problem 2(c)

$\int_0^{10} H'(t)\,dt = H(t)$ for $0 \le t \le 10$

$\frac{43-66}{10-0} = -2.3$

This is what the temperature in °C should be decreasing at if it was decreasing at a constant °C / minute.

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Work for problem 2(d)

tea at $t=10$ is 43°C

$= \int -13.84e^{-.173t}$

$0 = \int -13.84e^{-.173(0)} + C$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 2

Overview

In this problem students were presented with a table giving Celsius temperatures $H(t)$ of a cooling pot of tea during selected times between $t = 0$ and $t = 10$ minutes. Part (a) asked for an approximation for the rate of change of the tea's temperature at time $t = 3.5$. Students needed to construct a difference quotient using the temperature values across the smallest time interval containing $t = 3.5$ that is supported by the table. Part (b) asked for an interpretation of $\frac{1}{10}\int_0^{10} H(t)\,dt$ and a numeric approximation to this expression using a trapezoidal sum with the four intervals indicated by the table. Students should have recognized this expression as providing the average temperature of the tea, in degrees Celsius, across the time interval $0 \le t \le 10$ minutes. Part (c) asked for an evaluation of and interpretation of $\int_0^{10} H'(t)\,dt.$ Students needed to apply the Fundamental Theorem of Calculus and use values from the table to compute $H(10) - H(0).$ In part (d) students were told about biscuits that were removed from an oven at time $t = 0.$ It is given that the biscuits' temperature was $100°\text{C}$ initially, and that a function $B(t)$ modeling the temperature of the biscuits has derivative $B'(t) = -13.84e^{-0.173t}.$ Students were asked how much cooler the biscuits are than the tea at time $t = 10$ minutes. This was answered by taking the difference between the tea's temperature, $H(10),$ as supplied by the table, and the biscuits' temperature, $B(10),$ computed by $B(10) = 100 + \int_0^{10} B'(t)\,dt.$

Sample: 2A
Score: 9

The student earned all 9 points.

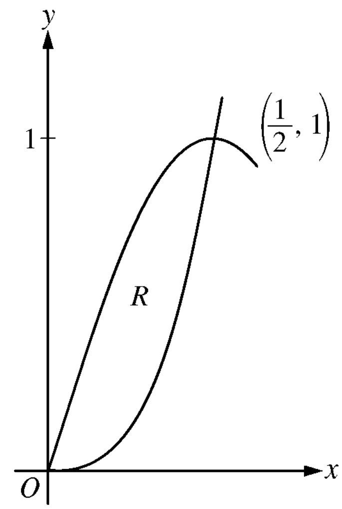
Sample: 2B
Score: 6

The student earned 6 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 3 points in part (d). In parts (a), (c), and (d) the student's work is correct. Units are not required in part (a) but are required in part (c). In part (b) the student's explanation is inadequate. The student appears to be using the Trapezoidal Rule rather than a general trapezoidal sum.

Sample: 2C
Score: 4

The student earned 4 points: 1 point in part (a), 2 points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student refers to "the average rate of change in temperature" rather than the average temperature. The student presents a correct trapezoidal sum and estimate, so the second and third points were earned in part (b). In part (c) the student's value is incorrect and the explanation contains no reference to the time interval. In part (d) the student's integral earned the first point.

Question 3



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$

(b)
$$\begin{aligned} \text{Area} &= \int_0^{1/2} (g(x) - f(x))\, dx \\ &= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx \\ &= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2} \\ &= -\frac{1}{8} + \frac{1}{\pi} \end{aligned}$$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c)
$$\begin{aligned} &\pi\int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx \\ &= \pi\int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx \end{aligned}$$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

NO CALCULATOR ALLOWED

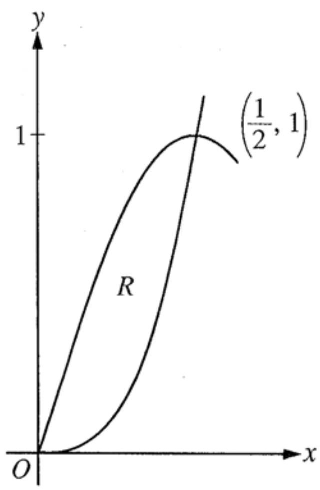
CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



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Work for problem 3(a)

$f(x) = 8x^3$

$f'(x) = 24x^2$

$f\left(\frac{1}{2}\right) = 8\left(\frac{1}{8}\right) = 1$ point of tangency : $\left(\frac{1}{2}, 1\right)$

$f'\left(\frac{1}{2}\right) = 24 \cdot \frac{1}{4} = 6$

$y = 6x + b$

$1 = 6\left(\frac{1}{2}\right) + b$

$1 = 3 + b$

$b = -2$

$y = 6x - 2$

The tangent line to the graph of f at $x = \frac{1}{2}$ is $y = 6x - 2$.

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Continue problem 3 on page 9.

Work for problem 3(b)

$f(x) = g(x)$

$8x^3 = \sin(\pi x)$

$x = 0, \frac{1}{2}$

let $u = \pi x$

$$R = \int_0^{\frac{1}{2}} [g(x) - f(x)]\,dx = \int_0^{\frac{1}{2}} [\sin(\pi x) - 8x^3]\,dx = \frac{1}{\pi}\int_0^{\frac{1}{2}} \sin u\,du - \int_0^{\frac{1}{2}} 8x^3\,dx$$

$$= \frac{1}{\pi}(-\cos(\pi x))\Big]_0^{\frac{1}{2}} - 2x^4\Big]_0^{\frac{1}{2}}$$

$$= \frac{1}{\pi}\left(-\cos\left(\frac{\pi}{2}\right) + \cos(0)\right) - 2\left(\frac{1}{2}\right)^4 + 0$$

$$= \frac{1}{\pi}(1) - \frac{2}{16}$$

$$= \frac{1}{\pi} - \frac{1}{8} = \boxed{\frac{8-\pi}{8\pi}}$$

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Work for problem 3(c)

$$\pi\int_0^{\frac{1}{2}} \left[(1 - f(x))^2 - (1 - g(x))^2\right] dx$$

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NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

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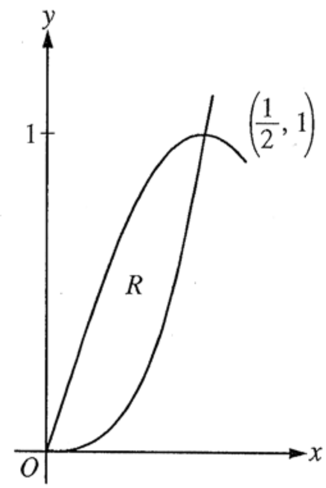

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Work for problem 3(a)

$f'(x) = 24x^2$

$24(1/2)^2 = 6$

$\boxed{y - 1 = 6(x - 1/2)}$

Continue problem 3 on page 9

NO CALCULATOR ALLOWED

Work for problem 3(b)

$$A = \int_0^{1/2} \left[\sin(\pi x) - 8x^3\right] dx$$

Work for problem 3(c)

$$V = \pi \int_0^{1/2} \left[(1-8x^3)^2 - (1-\sin(\pi x))^2\right] dx$$

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NO CALCULATOR ALLOWED

3C,

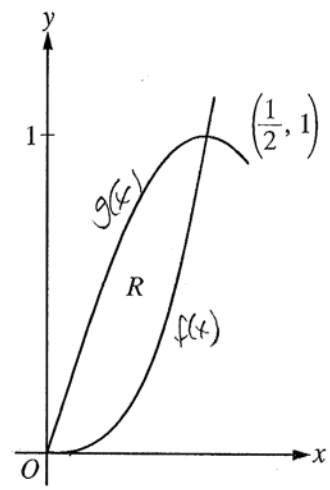
CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



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Work for problem 3(a)

$f(x) = 8x^3 \quad f'(x) = 24x^2$

when $x = 1/2 \quad f'(x) = 24\left(\frac{1}{4}\right) = 6$

$\boxed{y = 6x + b}$

Continue problem 3 on page 9

NO CALCULATOR ALLOWED

3C2

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Work for problem 3(b)

$A = \int_0^{1/2} \sin(\pi x) - 8x^3 \, dx$

Work for problem 3(c)

$\pi \int_0^{1/2} (\sin(\pi x) - 8x^3) \, dx = \text{Volume}$

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Question 3

Overview

This problem involved the graphs of functions $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$ that enclose a region R in the first quadrant. A figure depicting R was supplied, with the label $\left(\frac{1}{2}, 1\right)$ at the point of intersection of the graphs of f and g. Part (a) asked for an equation of the line tangent to the graph of f at $x = \frac{1}{2}$. Part (b) asked for the area of R, which required students to set up and evaluate an appropriate definite integral. For part (c) students needed to provide an integral expression for the volume of the solid that is generated when R is rotated about the horizontal line $y = 1$.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

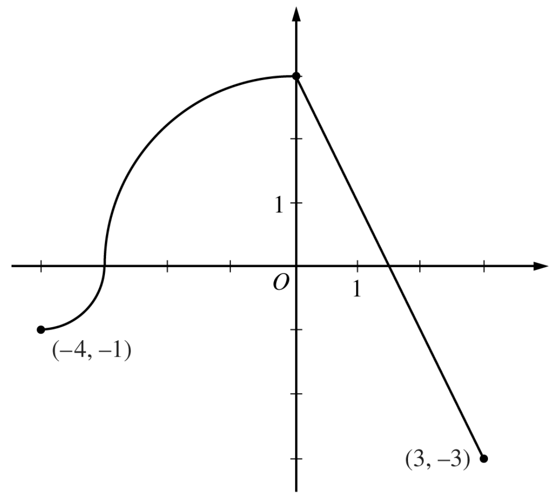
The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). In parts (a) and (c) the student's work is correct. In part (b) the student earned the integrand point only; the student does not find either antiderivative.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student earned the point for $f'\left(\frac{1}{2}\right)$. The student does not find the y-intercept of the tangent line. In part (b) the student earned the integrand point. In part (c) the student earned the limits and constant point.

AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 4



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$ $g'(x) = 2 + f(x)$ $g'(-3) = 2 + f(-3) = 2$	$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$
(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$. $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$. Therefore g has an absolute maximum at $x = \frac{5}{2}$.	$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$
(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.	1 : answer with reason
(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$. To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.	$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

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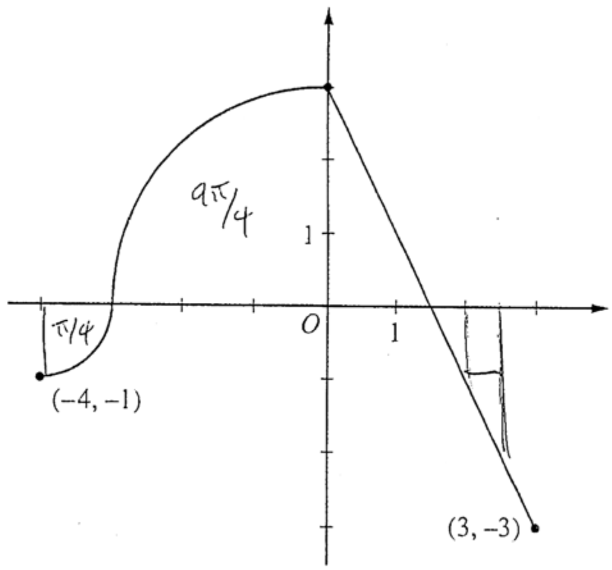


Graph of f

$\frac{\pi}{4}$ $\frac{\pi r^2}{4}$ $\frac{9\pi}{4}$

Work for problem 4(a)

$g(-3) = 2\cdot(-3) + \int_0^{-3} f(t)\,dt = \underline{-6 - \frac{9\pi}{4}}$

$g'(x) = \frac{d}{dx}\left(2x + \int_0^x f(t)\,dt\right) = \underline{2 + f(x)}$

$g'(-3) = 2 + f(-3) = 2 + 0 = \underline{2}$

Work for problem 4(b)

$g'(x) = 0$ $\quad 2 + f(x) = 0$

$f(x) = -2$

$x = 5/2$

$g'(x)$

$+ + + \;|\; - - - -$

$5/2$

check endpoints

$g(-4) = -8 + \int_0^{-4} f(t)\,dt$

$= -8 - 2\pi$

$g(5/2) = 5 + \int_0^{5/2} f(t)\,dt = 5 + \frac{5}{4}$

$g(3) = 6 + \int_0^3 f(t)\,dt = 6$

$x = 5/2$, because g' going from + to − proves it as the only relative maximum and $g(5/2)$ is greater than g at either endpoint.

Continue problem 4 on page 11

Work for problem 4(c)

$g''(x) = \frac{d}{dx}(g(x)) = f'(x)$

$f'(x)$

$+ + + + + + \mid - - - - -$

$-4 \quad -3 \quad 0 \quad 3/2 \quad 3$

The only point of inflection for g is at $x = 0$, since $f'(x)$, which is equivalent to g'', only changes signs at $x = 0$ on the interval $-4 \le x \le 3$

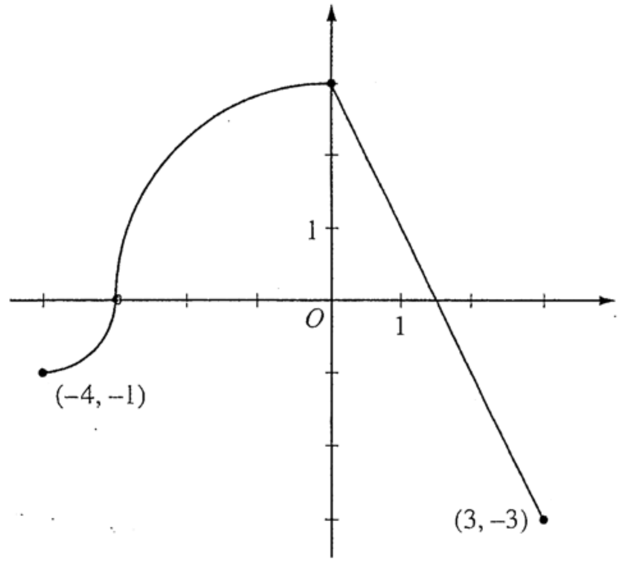
Work for problem 4(d)

Avg. Rate of change $= \dfrac{f(3) - f(-4)}{3 - -4}$

$= \dfrac{-3 - -1}{3+4} = \dfrac{-2}{7}$

Because Mean Value Theorem only applies when the function is continuous AND differentiable on the interval, which doesn't apply here since $f(x)$ isn't differentible at $x = 0$.

NO CALCULATOR ALLOWED



Graph of f

Work for problem 4(a)

a. $g(-3) =$

πr^2

$g(-3) = 2(-3) + \int_{-3}^{0} f(t)\,dt$

-6

$\frac{\pi}{4}$

It's the area of a circle w/ the radius of 3, ~~and~~ divided by 4.

$\boxed{g'(x) = 2 + f(x)}$

$g'(-3) = 2 + 0$

$\boxed{g'(-3) = 2}$

$\frac{9\pi}{4}$

$-6 + \frac{9\pi}{4} =$

$\rightarrow \boxed{g(-3) = -6 + \frac{9\pi}{4}}$

Work for problem 4(b)

absolute maximum can be found by equating the derivative of g to zero

$g'(x) = 2 + f(x) = 0$

$-2 = f(t)$

g has an absolute maximum when $x = 2.5$.

Continue problem 4 on page

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Work for problem 4(c)

where g has a point of inflection =

when $g''(x)$ is zero.

$g(x) = 2 + f(x)$

$g''(x) = f'(x)$.

$0 = f'(x)$

so when the f is experiencing its max/min.

g has points of inflection when x is equal to -4, 0, and -3.

because

the point of inflection is found when the second derivative is equal to zero (or changes signs).

$g''(x)$ ended up to be $f'(t)$. ~~because~~ I equated $f'(t)$ ~~to~~ zero, when the functions would be experiencing its points of inflection, the second. deriv. = 0.

∴ I found x value to be -4, 0, and 3.

Work for problem 4(d)

the average rate of change

$$\frac{f(3) - f(-4)}{3 - -4} = \frac{-3 + +1}{7} = \frac{-2}{7}$$

the average rate of change is $\frac{-2}{7}$

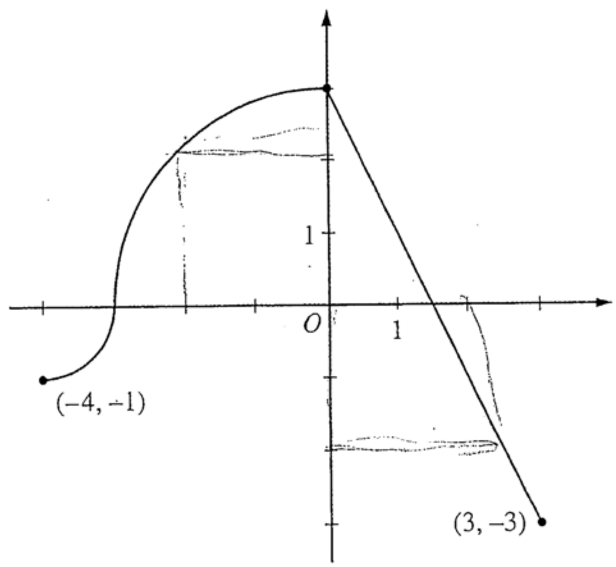
the mean value theorem state

1. if all value of x ~~are~~ is differentiable in the function, f in the closed interval [a,b]
2. if all value of x in the function, f is continuous in the open interval (a,b)

then there exits a value $f'(c) = \frac{f(b) - f(a)}{b-a}$.

However, f does not meet the requirements of the mean value theorem because it is not differentiable when x=0.

NO CALCULATOR ALLOWED



Graph of f

Work for problem 4(a)

$g(x) = 2(x) + \int_0^x f(t)\,dt$

$g(-3) = -6 + \frac{1}{4}\pi(3)^2$

$\boxed{g(-3) = -6 + \frac{9}{4}\pi}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3)$

$\boxed{g'(-3) = 2}$

Work for problem 4(b)

max occurs where $g'(x) = 0$ and where $g'(x)$ changes sign from + to −

$g'(x) = 2 + f(x)$

$0 = 2 + f(x)$

$-2 = f(x)$

$\boxed{\text{at } x = 2}$ there's a max

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Continue problem 4 on page

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Work for problem 4(c)

$g''(x) = 0$ means there's an inflection point

$g''(x) = f'(x)$

$0 = f'(x)$

So spots where slope $f(x) = 0$ or DNE,

the only such spot occurs at $\boxed{x=0}$

and at $\boxed{x=-3}$ where it's undefined

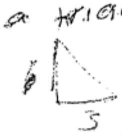
Work for problem 4(d)

avg rate of change $= \frac{1}{3-(-4)} \int_{-4}^{3} f(x)\,dx$

~~2 quarter circles~~

2 quarter circle

1 a triangle



$\frac{1}{7}\left[\frac{1}{4}\pi + \frac{9}{4}\pi + 9\right]$

$A = \frac{1}{2}(3)(6)$

$A = 9$

$\frac{1}{7}\left[\frac{10}{4}\pi + 9\right]$ = avg rate of change

Question 4

Overview

This problem provided the graph of a continuous function f, defined for $-4 \le x \le 3$. The graph consisted of two quarter circles and one line segment. The function g is defined by $g(x) = 2x + \int_0^x f(t)\, dt$. Part (a) asked for $g(-3)$, an expression for $g'(x)$, and the value of $g'(-3)$. These items tested the interpretation of a definite integral in terms of the area of a region enclosed by the x-axis and the graph of the function given in the integrand, as well as the application of the Fundamental Theorem of Calculus to differentiate a function defined by an integral with a variable upper limit of integration. Part (b) asked for the x-coordinate of the point at which g attains an absolute maximum for $-4 \le x \le 3$. Several approaches were possible, but they all begin with identification of candidates using the expression for $g'(x)$ found in part (a). Part (c) asked for locations of points of inflection for the graph of g, involving another analysis of $g'(x)$. Part (d) asked for the average rate of change of f on $-4 \le x \le 3$, and tested knowledge of the hypotheses of the Mean Value Theorem to explain why that theorem is not contradicted given the fact that its conclusion does not hold for f on $-4 \le x \le 3$.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no point in part (c), and 2 points in part (d). In part (a) the student makes a sign error in evaluating $g(-3)$ but correctly determines $g'(x)$ and evaluates $g'(-3)$, thus earning 2 of the 3 points. In part (b) the student earned the first 2 points for considering where $g'(x) = 0$ and correctly identifying 2.5 as the interior candidate for the x-coordinate of the absolute maximum. The student does not justify this as giving the absolute maximum, and so the final point in part (b) was not earned. In part (c) the student gives incorrect x-coordinates for the point of inflection. In part (d) the student's work is correct.

Sample: 4C
Score: 3

The student earned 3 points: 2 points in part (a), 1 point in part (b), no point in part (c), and no points in part (d). In part (a) the student makes a sign error in evaluating $g(-3)$ but correctly determines $g'(x)$ and evaluates $g'(-3)$, thus earning earned 2 of the 3 points. In part (b) the student earned the first point for $g'(x) = 0$. The student solves the equation incorrectly. In part (c) the student gives an incorrect x-coordinate for the point of inflection. In part (d) the student does not correctly compute the average rate of change and does not provide an explanation for why the Mean Value Theorem does not apply.

Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left.\frac{dW}{dt}\right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411$ tons

$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$\int \frac{1}{W - 300}\, dW = \int \frac{1}{25}\, dt$

$\ln|W - 300| = \frac{1}{25}t + C$

$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$

$W - 300 = 1100e^{\frac{1}{25}t}$

$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

5A1

NO CALCULATOR ALLOWED

Work for problem 5(a)

$25\overline{)1100}$ 44

at $t=0$, $W=1400$

so

$\left.\frac{dW}{dt}\right|_{t=0} = \frac{1}{25}(1100) = 44$ tons/year $= W'(0)$

$W(x+a) \approx W(x) + aW'(x)$

$W(0+\frac{1}{4}) \approx W(0) + \frac{1}{4}W'(0)$

$W(\frac{1}{4}) \approx 1400 + 11$

$W(\frac{1}{4}) \approx 1411$ tons

There will be about 1411 tons of landfill after 3 months.

Work for problem 5(b)

$\frac{dW}{dt} = \frac{1}{25}W - 12$

$\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt}$

$\boxed{\frac{d^2W}{dt} = \frac{1}{625}(W-300)}$

so $\frac{d^2W}{dt^2}$ is always positive b/c $W > 300$.

The answer in part a is an $\boxed{\text{underestimate}}$ because since $\frac{d^2W}{dt^2}$ is always positive for $t \geq 0$, the graph of W is concave up, so the linearization of W is an underestimate.

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Continue problem 5 on page 13.

5A2

Work for problem 5(c)

$$\frac{dW}{dt} = \frac{1}{25}(W-300)$$

$$\int \frac{dW}{W-300} = \int \frac{1}{25}\,dt$$

$$\ln|W-300| = \frac{1}{25}t + c$$

e e

can remove abs value (incr fn) b/c $W > 300$.

$$\rightarrow |W-300| = Ce^{\frac{1}{25}t}$$

Initial condition $W(0)=1400$

$$1400-300 = Ce^{0}$$

$$1100 = C$$

$$W-300 = 1100e^{\frac{1}{25}t}$$

$$\boxed{W = W(t) = 1100e^{\frac{1}{25}t} + 300}$$

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NO CALCULATOR ALLOWED

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Work for problem 5(a)

$\frac{dW}{dt} = \frac{1}{25}(1400-300) = \frac{1100}{25} \cdot \frac{1}{4} = \frac{1100}{100} = 11 + 1400 =$

$\boxed{1411 \text{ tons}}$

Work for problem 5(b)

$\frac{d^2W}{dt^2} = \frac{1}{25}W$

$\boxed{\text{1411 tons is an underestimate}}$

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Continue problem 5 on page 13.

5B2

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Work for problem 5(c)

$$\frac{dW}{dt} = \frac{1}{25}(W-300)$$

$$\int \frac{dW}{W-300} = \int \frac{1}{25}\,dt$$

$$e^{\ln|W-300|} = e^{\frac{1}{25}t + C}$$

$$W-300 = Ce^{\frac{1}{25}t}$$

$$W = Ce^{\frac{1}{25}t} + 300$$

$$1400 = Ce^{0} + 300$$

$$1400 = C + 300$$

$$C = 1100$$

$$\boxed{W = 1100e^{\frac{1}{25}t} + 300}$$

5C1

NO CALCULATOR ALLOWED

Work for problem 5(a)

$\frac{dW}{(W-300)} = \frac{1}{25}dt$

$\ln(W-300) = \frac{1}{25}t$

$W-300 = e^{\frac{t}{25}}$

$W = e^{\frac{t}{25}} + 300 + 1400$

$W = e^{\frac{t}{25}} + 1700$

at ~~t = 1/4 the amount of waste~~ is $e^{\frac{25}{4}} + 1700$

waste amount is $e^{\frac{25}{4}} + 1400$

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Work for problem 5(b)

$\frac{d^2W}{dt^2} = \frac{1}{25}\left(\frac{dW}{dt} - 0\right)$

overestimate

$\frac{1}{25}W - \frac{300}{25}$

$\frac{dW}{dt} = \frac{1}{25}W - 12$

$\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} - 0$

$\frac{d^2W}{dt^2} \frac{1}{25}\left(\frac{1}{25}(W-300)\right)$

$\frac{d^2W}{dt^2} = \frac{1}{625}(W-300)$

625

25
25
125
50

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Continue problem 5 on page 13.

Work for problem 5(c)

$$\frac{1}{25}dt = \frac{dW}{(w-300)}$$

$$\frac{1}{25}t = \ln(w-300)$$

$$e^{t/25} = w-300$$

$$w = e^{t/25}+300+C$$

$$1400 = e^{0/25}+300+C$$

$$1400 = 301+C$$

$$1099 = C$$

$$w = e^{t/25}+301+1099$$

$$w = e^{t/25}+1400$$

301

00

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Question 5

Overview

The context of this problem was accumulating waste at a landfill. The landfill contained 1400 tons of waste at the beginning of 2010, and a function W modeling the total tons of waste in the landfill satisfies $\frac{dW}{dt} = \frac{1}{25}(W - 300)$, where t is measured in years since the start of 2010. Part (a) asked for an approximation to $W\left(\frac{1}{4}\right)$ using a tangent line approximation to the graph of W at $t = 0$. Part (b) asked for $\frac{d^2W}{dt^2}$ in terms of W, and students should have used a sign analysis of $\frac{d^2W}{dt^2}$ to determine whether the approximation in part (a) is an overestimate or an underestimate. Part (c) asked students to solve the initial value problem $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with $W(0) = 1400$ to find $W(t)$. Students should have used the method of separation of variables.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 1 point in part (a), no points in part (b), and 5 points in part (c). In part (a) the student earned the first point. The student has the correct answer of 1411 but links it incorrectly with equal signs to 44. As a result of this error, the second point was not earned. In part (b) the student's work is incorrect. In part (c) the student's work is correct.

Sample: 5C
Score: 3

The student earned 3 points: no points in part (a), 1 point in part (b), and 2 points in part (c). In part (a) the student's work is incorrect. In part (b) the student has a correct $\frac{d^2W}{dt^2}$. In part (c) the student earned the first 2 points for correct separation of variables and antidifferentiation. In part (c) absolute value symbols were not required for $\ln(W - 300)$ because $W > 1400$. The constant of integration appears late, so the student was not eligible for any additional points.

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Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim\limits_{x \to 0^-} (1 - 2\sin x) = 1$

$\lim\limits_{x \to 0^+} e^{-4x} = 1$

$f(0) = 1$

So, $\lim\limits_{x \to 0} f(x) = f(0)$.

Therefore f is continuous at $x = 0$.

2 : analysis

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$-2\cos x \neq -3$ for all values of $x < 0$.

$-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$.

Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.

$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

(c)
$$\begin{aligned}
\int_{-1}^{1} f(x)\,dx &= \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx \\
&= \int_{-1}^{0} (1 - 2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx \\
&= \Big[x + 2\cos x\Big]_{x=-1}^{x=0} + \left[-\frac{1}{4}e^{-4x}\right]_{x=0}^{x=1} \\
&= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)
\end{aligned}$$

$$\begin{aligned}
\text{Average value} &= \frac{1}{2}\int_{-1}^{1} f(x)\,dx \\
&= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}
\end{aligned}$$

$4 : \begin{cases} 1 : \int_{-1}^{0} (1 - 2\sin x)\,dx \text{ and } \int_{0}^{1} e^{-4x}\,dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED

6A₁

Work for problem 6(a)

To be continuous

i) $f(0)=1$

ii) $\lim_{x\to 0^-} 1-2\sin x = \lim_{x\to 0^+} e^{-4x}$

$1 = 1 \quad \therefore \lim_{x\to 0^-} f = \lim_{x\to 0^+} f$

$\therefore \lim_{x\to 0} f = 1$

iii) $f(0) = \lim_{x\to 0} f = 1$

$\therefore$ f is continuous for all values of x.

Work for problem 6(b)

$$f'(x) = \begin{cases} -2\cos x, & x<0 \\ -4e^{-4x}, & x>0 \end{cases}$$

$f'(x) = -3$

-Since $-2\cos x$ oscillates between -2 and 2 there will be no such value in this function such that $f'(x)=-3$

But $f'(x) = -3$

$-4e^{-4x} = -3$

$e^{-4x} = 3/4$

$-4x = \ln(\frac{3}{4}) \quad \therefore x = \frac{-1}{4}\ln(\frac{3}{4}) \quad \therefore f'(\frac{-1}{4}\ln(\frac{3}{4})) = -3$

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Continue problem 6 on page 15.

6A$_2$

Work for problem 6(c)

$$f_{avg} = \frac{1}{1+1} \int_{-1}^{1} f(x)\,dx$$

$$= \frac{1}{2}\left[\int_{-1}^{0} 1 - 2\sin x\,dx + \int_0^1 e^{-4x}\,dx \right]$$

$$= \frac{1}{2}\left[\left[x + 2\cos x\right]_{-1}^{0} + \left[\frac{-1}{4}e^{-4x}\right]_0^1 \right]$$

$$\frac{1}{2}\left[2 + 1 - 2\cos(-1) + \frac{-e^{-4}}{4} + \frac{1}{4} \right]$$

$$= \frac{\left[3 - 2\cos(-1)\right] + \left[\frac{-1}{4e^4} + \frac{1}{4}\right]}{2}$$

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6B1

Work for problem 6(a)

$1-2\sin x = e^{-4x}$

$e^{-4(0)} = 1$ and $1-2\sin(0) = 1-0 = 1$

They're both $=1$ @ $x=0$, therefore they're continuous.

Work for problem 6(b)

$$f'(x) = \begin{cases} -2\cos x & \text{for } x<0 \\ -4e^{-4x} & \text{for } x>0 \end{cases}$$

$-3 = -2\cos x$

$\frac{2}{3} = \cos x$

$\boxed{x = \cos^{-1}\frac{2}{3}}$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

6B₂

Work for problem 6(c)

$$\frac{1}{2}\left(\int_{-1}^{0} 1-2\sin x\,dx + \int_{0}^{1} e^{-4x}\,dx\right)$$

$$\left(\left(x+2\cos x\right)\Big|_{-1}^{0}\right)+\left(-\frac{1}{4}e^{-4x}\Big|_{0}^{1}\right)$$

$$\left(0+2\cos 0\right)-\left(-1+2\cos -1\right)+\frac{-1}{4e^{4}}+\frac{1}{4}$$

$$2+1-2\cos(-1)+\frac{-1}{4e^{4}}+\frac{1}{4}$$

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GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6C$_1$

Work for problem 6(a)

$1 - 2\sin x = e^{-4x}$

$1 - 2\sin(0) = e^{-4(0)}$

$1 = 1$

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Work for problem 6(b)

$f'(x) = \begin{cases} -2\cos x \\ -4e^{-4x} \end{cases}$

$x = -\dfrac{\ln\frac{3}{4}}{4}$

$-4e^{-4x} = -3$

$e^{-4x} = \frac{3}{4}$

$-4x = \ln\frac{3}{4}$

Do not write beyond this border.

Continue problem 6 on page 15.

6C2

Work for problem 6(c)

$$\frac{f(1)-f(-1)}{1+1} = \frac{e^{-4}-2\sin 1-1}{2}$$

Question 6

Overview

This problem defined the function f using one expression for $x \le 0$ and a different expression for $x > 0$. Part (a) asked whether f is continuous at $x = 0$. Students needed to acknowledge that the left- and right-hand limits as $x \to 0$ and the value $f(0)$ all agree. Part (b) asked for a piecewise expression for $f'(x)$ and the value of x for which $f'(x) = -3$. This involves taking the symbolic derivatives of the branches of f and recognizing which piece produces a value of -3. Part (c) asked for the average value of f on the interval $[-1, 1]$. The required integral must be split at 0 to use the antiderivatives of the two branches of f.

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), and 3 points in part (c). In part (a) the student begins the analysis of continuity by looking at the functional values on each side of 0. The student does not use limits and does not consider $f(0) = 1$, thus earning only 1 of the possible 2 points. In part (b) the student presents a correct piecewise derivative, so the first 2 points were earned. The student's value of x is incorrect. In part (c) the student earned the first 3 points. The student does not multiply by $\frac{1}{2}$, so the answer point was not earned.

Sample: 6C
Score: 3

The student earned 3 points: 1 point in part (a), 2 points in part (b), and no points in part (c). In part (a) the student begins the analysis of continuity by looking at the functional values on each side of 0. The student does not use limits and does not consider $f(0) = 1$, thus earning only 1 of the possible 2 points. In part (b) the student does not give a correct piecewise presentation for $f'(x)$ and so earned 1 of the possible 2 points for $f'(x)$. The student finds the correct value of x and earned the third point in part (b). In part (c) the student's work is incorrect.